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Thermoeconomic analysis of an irreversible Stirling heat pump cycle

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Abstract. In this paper an analysis of the Stirling cycle in thermoeconomic terms is developed using the entropy generation. In the thermoeconomic optimization of an irreversible Stirling heat pump cycle the F function has been introduced to evaluate the optimum for the higher and lower sources temperature ratio in the cycle: this ratio represents the value which optimizes the cycle itself. The variation of the function F is proportional to the variation of the entropy generation, the maxima and minima of F has been evaluated in a previous paper without giving the physical foundation of the method. We investigate the groundwork of this approach: to study the upper and lower limits of F function allows to determine the cycle stability and the optimization conditions. The optimization consists in the best COP at the least cost. The principle of maximum variation for the entropy generation becomes the analytic foundation of the optimization method in the thermoeconomic analysis for an irreversible Stirling heat pump cycle.

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1 Introduction

The Stirling cycle is an important model of refrigeration systems and the recent developments in design were proposed after the new concept of finite time thermodynamics came into existence [1–3]. Blanchard applied the Lagrange multiplier method to find out the COP of an endoreversible Carnot heat pump operating at the minimum power input for a given heating load [4]. Several papers have been devoted to propose mathematical functions to optimize thermodynamics cycles starting from different initial conditions and focusing on the total cost and efficiency. The definition of optimization that we adopt in this paper is the best COP at the least cost. The performance of the different heat engines and refrigeration systems were investigated using the concept of finite time thermodynamics, of the ecological approach and the thermoeconomic analysis [1,5–7]. On the other hand, the key-role of entropy generation maximum has been recently demonstrated in thermodynamics analysis of the irreversible processes and it has been shown that it represents a new criterion for determining the conditions for stability [8–13]. In this paper we show that this criterion represents the thermodynamic foundation for some recent results obtained in thermoeconomic analysis of the Stirling heat pump cycle. We start from studying the time evolution of an open system and we take as working hypothesis that it evolves in the optimization of the irreversibilitiy due to entropy generation. F function is well suited to show the system evolution related to the irreversibility due to entropy generation, our aim is to propose an approach based upon the natural behaviour of the thermodynamics/thermoeconomic system as a groundwork for the optimization analysis. The evolution of an open system is considered natural when it moves in order to get the optimization of the entropy generation.

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2 The thermodynamic analysis

The working substance of the Stirling cycle may be a gas, a magnetic material, etc., and for different working fluids the performance of the cycle are quite different. The Stirling cycle with an ideal gas consists of two isothermal and two isochoric processes. It approximates the expansion stroke of the real cycle by an isothermal process to whom heat is added to reach the temperature T_c from a heat source of finite capacity whose temperature varies from T_{L1} to T_{L2}. The heat addition to the working fluid is thought as an isochoric process: heat is going towards the heat sink of finite heat capacity that gets a temperature variation from T_{H1} to T_{H2}. The heat rejection from the working fluid to the regenerator is modelled as an isochoric process which completes the cycle itself. Let Q_c and Q_h be the amount of heat absorbed from the sources at the temperature T_c and T_h respectively, during the two isothermal processes [1]:

$$Q_h = C_H \epsilon_H (T_h - T_{H1}) t_h \tag{1}$$

$$Q_c = C_L \epsilon_L (T_{L1} - T_c) t_L \tag{2}$$

where C_H is the heat capacitance rate of the sink reservoir, C_L is the heat capacitance rate of the source reservoir, t_H is the heat rejection time, t_L is the heat addition time, ϵ_H is the effectiveness of the heat exchangers for the hot-side and ϵ_L is the effectiveness of the heat exchangers for the cold-side. These cycles do not possess the condition of perfect regeneration, hence it is assumed that the loss per cycle, ΔQ_R, is proportional to the temperature difference of the two isothermal processes as follows [1,14–18]:

$$\Delta Q_R = n c_f (1 - \epsilon_R)(T_h - T_c) \tag{3}$$

where c_f is the molar heat capacity of the working fluid and n is the number of moles. The Gouy-Stodola theorem [19] states that the thermodynamic work burnt in the irreversibility due to the entropy generation is equal to the product between the lowest source temperature and the entropy generation, i.e. total entropy is equal to the isolated system entropy plus the irreversibility due to entropy generation. Considering the Gouy-Stodola theorem and the definition of the entropy due to irreversibility ΔS_{irr} [12], the last one can be written as:

$$\begin{aligned}\Delta S_{irr} &= \frac{\Delta Q_R}{T_c} \\ &= n c_f (1 - \epsilon_R) \frac{(T_h - T_c)}{T_c} = n c_f (1 - \epsilon_R)(x - 1)\end{aligned} \tag{4}$$

with $x = T_h/T_c$. The theorem of maximum entropy generation states that the entropy generation is maximum at stationary state [8]. This theorem allows a new approach to irreversible processes as it is proved in a lot of different applications in hydrodynamics [9], engineering thermodynamics [10], rational thermodynamics [11] and biophysics [12,13]. Hence applying it here, we argue that equation 4 must be a maximum in the thermodynamics stability: this equation described the natural behavior of the thermodynamics system.

3 The thermodynamics foundation of the thermoeconomic analysis

The objective function F of the thermoeconomic optimization recently proposed is [1,20,21]:

$$F = \frac{\dot{Q}_H}{C_i + C_e} \tag{5}$$

with $\dot{Q}_H$ = heating power, C_i and C_e refer to annual investment and energy consumption costs, and are defined as:

$$C_i = a(A_H + A_L + A_R) + b \frac{Q_h - Q_c}{t_{cycle}} \tag{6}$$

$$C_e = b \frac{Q_h - Q_c}{t_{cycle}} \tag{7}$$

where a is a constant directly proportional to the investment cost of the heat exchanger and is equal to the capital recovery factor multiplied by the investment cost per unit heat exchanger area. $A_H + A_L + A_R$ is the heat exchanger total area, with A_H the heating area, A_L the heat source area and A_R the regenerative area. b is the capital recovery factor multiplied by the investment cost per unit power input and t_{cycle} is defined as:

$$t_{cycle} = t_H + t_L + t_R \tag{8}$$

with

$$t_R = 2\alpha(T_h - T_c) = 2\alpha T_c (x - 1) \tag{9}$$

where α is a constant that depends upon the kind of working fluid used in the cycle, and shows that the working time of the regenerator (a sort of recovering time towards the initial conditions in thermoeconomics) is proportional to the difference of temperature. In the thermoeconomic analysis of an irreversible Stirling heat pump cycle the function F has been used to evaluate the ratio of the higher and lower source temperature in order to reach the optimization of the cycle itself. The common solution, based upon the application of the variation method, consists in evaluating the maxima of F function, solving the equation $\delta F = 0$ [1] applying the variational method.

Now, from 6 and 7 the 5 becomes:

$$F = \frac{\dot{Q}_H}{a(A_H + A_L + A_R) + (b + b') \frac{Q_h - Q_c}{t_{cycle}}}. \tag{10}$$

Starting from the relations 4, 5, 8-10, we can argue that the objective function F of the thermoeconomic optimization is related to the entropy generation as follows:

$$F = \frac{\dot{Q}_H}{a(A_H + A_L + A_R) + (b + b') \frac{Q_h - Q_c}{t_H + t_L + \frac{2\alpha T_c}{n c_f (1 - \epsilon_R)} \Delta S_{irr}}} \tag{11}$$

which can be easily written after few algebric operations:

$$F = = \frac{\Gamma_1 + \Gamma_2 \Delta S_{irr}}{\Gamma_3 + \Gamma_4 \Delta S_{irr}} \tag{12}$$

$$\text{with} \quad \begin{cases} \Gamma_1 = \dot{Q}_H (t_H + t_L) \\ \Gamma_2 = \frac{2\alpha T_c \dot{Q}_H}{n c_f (1 - \epsilon_R)} \\ \Gamma_3 = a(A_H + A_L + A_R)(t_H + t_L) \\ \qquad +(b - b')(Q_h - Q_c) \\ \Gamma_4 = \frac{2\alpha a (A_H + A_L + A_R) T_c}{n c_f (1 - \epsilon_R}. \end{cases}$$

From equation (12) we can argue that the variation of the function F is proportional to the variation of the entropy generation:

$$\delta F = \frac{\Gamma_2\Gamma_3 - \Gamma_1\Gamma_4}{\Gamma_3 + \Gamma_4 \Delta S_{irr}} \delta(\Delta S_{irr}) \tag{13}$$

with

$$\Gamma_2\Gamma_3 \neq \Gamma_1\Gamma_4. \tag{14}$$

Then it follows that

$$\delta(\Delta S_{irr}) = 0 \Rightarrow \delta F = 0. \tag{15}$$

In this way it has been stressed the relation between the economic analysis and the thermodynamics. In the economical analysis the function F was introduced in several papers: we need to know its upper and lower limits to fulfill the basic conditions of optimization, but no physical explanation has been up to now given about this method. Here we prove that the limits of the F function are directly correlated to the entropy generation in the state of stability and related to the optimization of the cycle. Hence the optimization, which consists in the best COP related to the least cost, can be obtained in the conditions of natural stability for the open systems. The evolution of an open system is defined natural when it moves to get the optimization of entropy. The advantages of this method consist in exploiting the natural dynamics of the system in order to reach, following its natural behaviour, the optimum by the shortest way (i.e. the lower cost).

4 Conclusions

The thermodynamic and thermoeconomic analysis of the optimization of an irreversible Stirling heat pump cycle is presented in relation with its thermodynamic foundation. We proved that the principle of maximum variation for the irreversible entropy is the analytic foundation for the optimization method recently introduced in the thermoeconomic analysis for an irreversible Stirling heat pump cycle. Of course it represents not only an analytical and mathematical groundwork, but also the physical and thermodynamic foundation for the method itself, as a consequence of the physical meaning of the principle of maximum entropy variation in thermodynamics [8–13]. The optimization method is a useful tool to design thermodynamics systems characterized by lower working costs. The principle of maximum variation allows a deeper thermoeconomic analysis focused on the stability conditions.

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